

Nearing the Horizon of a Heterotic String

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Abstract

It is argued that recent developments point to the existence of an $AdS_3 \times S^2 \times T^5$ holographic dual for the 2D CFT living on the worldsheet of N coincident heterotic strings in a T^5 compactification, which can in turn be described by an exact worldsheet CFT. A supergravity analysis is shown to imply that the global supergroup is $Os(4^*|4)$, with 16 supercharges and an affine extension given, surprisingly, by a *nonlinear* $\mathcal{N} = 8$ 2D superconformal algebra. Possible supergroups with 16 supercharges are also found to match the expected symmetries for T^n compactification with $0 \leq n \leq 7$.

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1. Introduction and summary

The worldsheet of N stretched, coincident heterotic strings is described by a $(c_L, c_R) = (24N, 12N)$ 2d CFT. General considerations as well as recent investigations, both described below, raise the intriguing possibility that this CFT has an $AdS_3 \times M$ holographic spacetime dual. If so, the first-quantized Hilbert space of the N stretched heterotic strings would be identified with the second-quantized Hilbert space of interacting closed heterotic strings on $AdS_3 \times M$. In this paper, as reported in [1], we continue these investigations and in particular find some surprising results about the structure of the supersymmetry group.

Why should we expect such a holographic dual? At strong coupling, the heterotic string becomes a $D1$ -brane of the type I theory. General low-energy scaling arguments coupled with open-closed duality then suggest the existence of a holographic dual. Because the low-energy limit of the worldvolume theory is conformally invariant, the dual should contain an AdS_3 factor. In addition, N stretched heterotic strings have an exponentially growing spectrum of left-moving BPS excitations. Although the growth is not enough to make a black string with a horizon large compared to the string scale, it is still the case that in the classical limit,¹ the second law of thermodynamics forbids energy from leaking

¹ With the spacetime momentum density along the string, and all spacetime fields fixed while $\hbar \rightarrow 0$.

off of the N strings, just as it does for a large black hole. One expects this behavior to be explained in the macroscopic spacetime picture by the appearance of a stringy horizon and associated near-horizon scaling solution. However, the real situation is likely more subtle than these general comments indicate. As we shall see below, simple group theory implies that the situation is highly dimension-dependent. In particular, concrete indicators of a holographic dual in the special case of compactification to $D = 5$, on which we largely concentrate, will be reviewed below.

1.1. The leading-order solution

The string frame classical geometry sourced by the N stretched heterotic strings in the leading α' approximation for a compactification to $D \geq 5$ dimensions was found some time ago [2] using the supergravity equations:

$$ds^2 = \frac{dx^+ dx^-}{1 + N\left(\frac{r_h}{r}\right)^{D-4}} - d\vec{x} \cdot d\vec{x} - ds_{10-D}^2, \quad (1.1)$$

where $r_h^{D-4} = \frac{g_{10}^2}{8\pi^5 V_{10-D}}$, \vec{x} is a transverse $D - 2$ vector and $r^2 = \vec{x} \cdot \vec{x}$. The string coupling behaves as

$$e^{2\Phi} = \frac{e^{2\Phi_0}}{1 + N\left(\frac{r_h}{r}\right)^{D-4}}, \quad (1.2)$$

and there is also a Kalb-Ramond field

$$H = dx^+ dx^- de^{2(\Phi - \Phi_0)}. \quad (1.3)$$

This spacetime is singular at the core of the string $r = 0$. Interestingly the string coupling goes to zero while the curvature diverges. This suggests the possibility that the singularity might be resolved within classical string theory by α' corrections.

1.2. Small 4d black holes and small 5d black strings

Recently there have been compelling indicators [3-10] that such a stringy resolution in fact occurs for the case $D = 5$. The story began with an S^1 compactification from 5 to 4 dimensions, in which the stretched strings are wrapped around the S^1 and become a pointlike object in $D = 4$. This “small black hole” has BPS excitations with momentum-winding (N, k) and a degeneracy that grows at large charges as $e^{4\pi\sqrt{Nk}}$. As above, this growth is not rapid enough to make a large black hole visible in supergravity, for which it is easily seen that the entropy must scale as the square of the charges. Nevertheless,

the charges of the small black hole were plugged into the entropy formula derived in an α' expansion for large black holes and found to reproduce – to all orders! – the known BPS degeneracies [3-5].

This impressive agreement was surprising because the macroscopic derivation of the entropy as a function of the charges employs the known spacetime black hole attractor geometry [11] as an intermediate step. For small black holes, no such solutions were known. Subsequently, it was found that when stringy R^2 corrections to the supergravity equations are included, solutions with string-scale horizon do exist, and furthermore the horizon area scales in the right way with the charges [6]. Of course such solutions can only be regarded as suggestive, due to the ambiguities arising from field redefinitions and uncontrolled effects of R^4 and higher corrections. Nevertheless, the remarkable coherence of the small black hole picture suggests that we take them seriously. Ultimately the existence or not of these solutions should be addressed using worldsheet CFT methods, which can control all the α' corrections.

1.3. Small black strings

The near-horizon geometry of the small black holes contains an AdS_2 factor with an electric field associated to the Kaluza-Klein $U(1)$. Hence we have an S^1 fibered over the AdS_2 . The total space of such a bundle is a quotient of AdS_3 . Taking the cover of this quotient, we obtain the AdS_3 factor of the near horizon geometry of N stretched heterotic strings in 5 dimensions. Indeed the full 5d solutions were seen directly in T^5 compactification to 5 dimensions in a recent elegant paper CDKL (Castro, Davis, Kraus and Larsen) [8]. CDKL begin with the R^2 -corrected $D = 5$ $\mathcal{N} = 2$ supersymmetry transformation laws and find half BPS solutions with a near-horizon $AdS_3 \times S^2$ factor and charges corresponding to N heterotic strings. In the heterotic frame, these solutions have a string coupling proportional to $\frac{1}{\sqrt{N}}$.

1.4. The near-horizon nonlinear superconformal group

The symmetries of the near-horizon region of this solution are of special interest. One expects the number of supersymmetries in the near horizon region to double from 8 to 16. But there are only four Lie superalgebras of classical type with 16 supercharges and an $SL(2, \mathbf{R})$ factor: $Osp(4^*|4)$, $SU(1,1|4)$, $F(4)$, and $Osp(8|2)$, with bosonic R-symmetry factors $SU(2) \times Sp(4)$, $SU(4) \times U(1)$, $SO(7)$, and $SO(8)$, respectively.² This

² See for example [12]. One may also consider the product group $D(2,1;\alpha) \times D(2,1;\alpha)$.

is puzzling for two reasons. Firstly, R-symmetries usually arise geometrically as spacetime isometries – *e.g.* the $SO(6)$ R-symmetry corresponding to S^5 rotations of the near horizon D3 geometry. But in the current context that gives at most the $SU(2)$ rotations of the S^2 and so cannot account for the R-symmetry of any of the above supergroups. Secondly, as shown by Brown and Henneaux [13], when there is an AdS_3 factor the superisometry group must have an affine extension containing a Virasoro algebra. However there are no linear “ $\mathcal{N} = 8$ ” superconformal algebras containing any of the above superalgebras as global subalgebras.

We show herein by direct computation that the global superisometry group is in fact $Osp(4^*|4)$. As usual, the $SU(2)$ factor of the R-symmetry arises from the geometric rotational isometries of the S^2 R-symmetry. From the 5d point of view, the $Sp(4) \sim SO(5)$ arises unusually from the global $Sp(4)$ R-symmetry of 5d $\mathcal{N} = 4$ supergravity. From the 10d point of view, these are the $SO(5)$ rotations of the spin frame for the T^5 . This $SO(5)$ acts only on fermions and is not to be confused with spacetime rotations.

While this explains how the large global R-symmetry arises, it does not explain the puzzle with the affine extension. While there are no linear superconformal algebras with more than 4 supercurrents (which means 8 global supercharges in the NS sector), there are a few *nonlinear* algebras with 8 supercurrents. These were classified some time ago by [14,15,16], and one of these algebras, let us denote it $\hat{Osp}(4^*|4)$, indeed contains $Osp(4^*|4)$ in the $k \rightarrow \infty$ limit. The nonlinearity in the commutation relation is confined to the commutator of the supercurrents, which takes the schematic form

$$\{G_r^I, G_s^J\} \sim 2\delta^{IJ}L_{r+s} + (r-s)(R^{IJ})_{r+s} + \sum_p (R^I_K)_{r+s-p}(R^{KJ})_p + \dots, \quad (1.4)$$

where the current R^{IJ} generates the bosonic R-symmetry group. The nonlinear superconformal algebras are a special type of W algebra with only one spin two current and no higher currents. Though known for some time [17], these algebras have not seen many applications in string theory or elsewhere. We note that it is not fully understood when these algebras have unitary representations.

Fortuitously, consistent boundary conditions on AdS_3 with more than eight global supersymmetries and their associated asymptotic symmetry algebras were studied in [18]. The short list contains $\hat{Osp}(4^*|4)$. We conclude that the near-horizon symmetry algebra of the R^2 -corrected supergravity solutions corresponding to N stretched heterotic strings is $\hat{Osp}(4^*|4)$.

1.5. $D \neq 5$

It is interesting to see how or if a picture could emerge in dimensions other than 5 consistent with the the known supergroups. Near-horizon symmetry enhancement suggests that there should always be 16 near-horizon supersymmetries.³ In $D = 10$, it is natural to speculate that there is a stretched-string solution with an $AdS_3 \times S^7$ near horizon region with the $Osp(8|2)$ superisometry group and a geometrically realized $SO(8)$ R-symmetry. In $D = 9$, $F(4)$ could arise with a geometrical $SO(7)$. It could also arise in $D = 3$, but with a nongeometrical $SO(7)$ from spin frame rotations of the T^7 . In $D = 8$, the horizon is an S^5 , so we could have $SU(1,1|4)$ with the $SU(4) \sim SO(6)$ geometrically realized and the $U(1)$ nongeometrical. $SU(1,1|4)$ is also a candidate for $D = 4$ with a nongeometrical $SO(6)$ and the $U(1)$ realized geometrically as rotations of the S^1 horizon. In $D = 7$, the horizon is an S^4 so one could again have $Osp(4^*|4)$, but with a geometrical $Sp(4) \sim SO(5)$ and a nongeometrical $SU(2)$. In $D = 6$, which is the self-dual dimension for strings, the near horizon geometry would be $AdS_3 \times S^3 \times T^4$. This has both a geometrical and a nongeometrical $SO(4)$, both of which have $SU(2)$ subgroups. This could correspond to two copies of $D(2,1;\alpha)$ with left and right actions, each containing an $SU(2) \times SU(2)$ R-symmetry. So for all $3 \leq D \leq 10$, there are candidate near-horizon supergroups with 16 supercharges.⁴ Whether or not the solutions actually exist remains to be seen.

1.6. A worldsheet CFT?

As discussed above, the success of the small black hole/string story suggested that N heterotic strings in $D = 5$ have an $AdS_3 \times S^2$ near horizon region. The value of the string coupling goes to zero as $N \rightarrow \infty$ so that string loop corrections can be ignored. Such solutions were then found in the classical stringy R^2 -corrected supergravity, but R^2 -corrected supergravity is unreliable because α' corrections are uncontrolled. Such corrections, however, are controllable using worldsheet CFT methods, so either the authors of [6,7,8] as well as the authors of the present paper were misled by the solutions of R^2 -corrected supergravity, or an exact worldsheet CFT which describes the near-horizon geometry must exist.

³ 14 is another possibility, corresponding to the supergroups $G(3)$ with R-symmetry group G_2 or $Osp(7|2)$ with $SO(7)$.

⁴ Candidates for near-horizon supergroups of type II strings can be obtained by taking left and right copies of the above, except for the case $D = 6$.

The sought after worldsheet CFT cannot involve a RR background, as there are none in heterotic string theory. Furthermore, the large spacetime symmetry group places strong constraints on the worldsheet CFT [19,20,21,22]. So if this CFT and the associated GSO projection exist, it should be possible to find them. Related and in some cases partial proposals have already appeared in [23,24,25] as well as [26,27] which appeared as the present work was under submission. The closely related problem of finding the CFT which describes the S^2 horizon of a heterotic monopole was solved in [28]. We will review this construction and its application to the current problem in the last section.

Supposing the CFT does not exist for some or all cases, and we have simply been misled by the R^2 solutions, what are the possibilities? One is that there simply is no near horizon solution and that both supergravity and the exact classical string theory are singular at the core of the string. A second possibility, advocated in [24] (but at odds with the picture in CDKL), is that there is a smooth near horizon solution, but that some of the supersymmetries act trivially. Phenomena of this type are known in string theory. For example if we look at the magnetically charged black hole solutions of [28], for magnetic charge ± 1 the horizon part of the “throat” theory is trivial and $SO(3)$ spacetime rotations act trivially. Should this turn out to be the case we will need to understand in what sense the near-horizon spacetime is the holographic dual of the heterotic string CFT.

2. Near horizon analysis

In this section we explicitly demonstrate, by finding the unbroken supersymmetries and computing their commutators, that the superisometry group of the near-horizon region of a fundamental heterotic string in R^2 -corrected supergravity is $Osp(4^*|4)$. We employ the asymptotically flat solution of the BPS conditions found in CDKL. The CDKL analysis was in turn made possible by the recent supersymmetric completion of the relevant R^2 term in five dimensions [29], which descends from terms related to anomaly cancellation in the M-theory lift.

2.1. Supergravity in 5d

Five dimensional supergravity with $8n$ real supercharges, conventionally referred to as $\mathcal{N} = 2n$ supergravity, has an $Sp(2n)$ R-symmetry group with the supersymmetry parameter ϵ^i , $i = 1, \dots, 2n$, transforming in the **2n**. CDKL work in an offshell $\mathcal{N} = 2$ formalism (which greatly simplifies the computation), but their solution can be embedded

in an $\mathcal{N} = 4$ theory as follows. The $\mathcal{N} = 4$ gravitino variation has relevant terms of the form

$$\delta\psi_\mu^i \sim \nabla_\mu \epsilon^i + (F_{\rho\sigma}^{ij} + G_{\rho\sigma} \Omega^{ij})(\gamma_\mu^{\rho\sigma} - 4\delta_\mu^\rho \gamma^\sigma) \epsilon_j + \dots, \quad (2.1)$$

where F^{ij} and G are 2-form field strengths in the **5** and **1** of $Sp(4)$ respectively. However, one can see upon dimensional reduction from $D = 10$ that the F^{ij} come from components of the metric g and anti-symmetric 2-form B which are mixed between $AdS_3 \times S^2$ and T^5 , and these vanish in the present context. Under these circumstances, the $Sp(4)$ R symmetry is unbroken by the background and the $\mathcal{N} = 4$ variation looks exactly like that of $\mathcal{N} = 2$ but with $i = 1, \dots, 4$, instead of $i = 1, 2$.

In $4 + 1$ dimensions there is no ordinary Majorana condition, but one can impose a symplectic-Majorana condition via

$$\bar{\xi}^i = \xi_i^\dagger \gamma^{\hat{0}} = \xi^{iT} C, \quad (2.2)$$

where C is the charge conjugation matrix and tangent-space indices are hatted. We will also use the symplectic matrix Ω_{ij} to raise and lower indices by

$$\xi^i = \Omega^{ij} \xi_j \quad \xi_i = \xi^j \Omega_{ji}. \quad (2.3)$$

We choose a basis in which

$$\Omega_{12} = \Omega_{34} = -\Omega_{21} = -\Omega_{43} = 1. \quad (2.4)$$

The string solution in supergravity has $ISO(1,1) \times SO(3)$ isometry. It is convenient to choose lightcone coordinates along the string $x^\pm = x^0 \pm x^1$, and spherical coordinates r, θ, ϕ for the transverse directions. In conformity with CDKL, we take the tangent space metric to have signature $(+ - - -)$.

We work in a representation of the Clifford algebra with $\gamma^{\hat{0}}$ Hermitian, and the other $\gamma^{\hat{\mu}}$ anti-Hermitian. Consistent with this, we can choose $\gamma^{\hat{1}}$ to be real while the others are pure imaginary. The charge conjugation matrix $C = \gamma^{\hat{0}\hat{1}}$ satisfies

$$C\gamma^\mu C^{-1} = \gamma^{\mu T}. \quad (2.5)$$

As this is a non-chiral theory, we choose $\gamma^{\hat{0}\hat{1}\hat{r}\hat{\theta}\hat{\phi}} = 1$ where the indices are tangent space indices, raised and lowered by $-\eta_{\hat{0}\hat{0}} = \eta_{\hat{1}\hat{1}} = \eta_{\hat{r}\hat{r}} = \eta_{\hat{\theta}\hat{\theta}} = \eta_{\hat{\phi}\hat{\phi}} = -1$. Note that $\gamma^{\mu_1 \dots \mu_p}$ is always the antisymmetric combination divided by $p!$.

2.2. Killing spinors

The CDKL solution has an $AdS_3 \times S^2$ near horizon region with metric

$$ds^2 = \frac{r}{l} dx^+ dx^- - \frac{l^2}{r^2} dr^2 - l^2 d\Omega_2. \quad (2.6)$$

Choosing the vielbein

$$e^{\hat{+}}_+ = e^{\hat{-}}_- = \sqrt{\frac{r}{l}}, \quad e^{\hat{r}}_r = \frac{l}{r}, \quad e^{\hat{\theta}}_\theta = l, \quad e^{\hat{\phi}}_\phi = l \sin \theta, \quad (2.7)$$

the only non-zero components of the spin connection are

$$\omega_\phi^{\hat{\theta}\hat{\phi}} = \cos \theta, \quad \omega_+^{\hat{\theta}\hat{+}} = \omega_-^{\hat{\theta}\hat{-}} = \frac{1}{2} \sqrt{\frac{r}{l^3}}. \quad (2.8)$$

The Weyl multiplet of $5d \mathcal{N} = 2$ conformal supergravity contains an auxiliary 2-form $v_{\mu\nu}$ (related to G in (2.1)) which is $v_{\hat{\theta}\hat{\phi}} = \frac{3}{4l}$ in this background. In terms of v the precise version of the gravitino variation (2.1) is

$$\delta_\epsilon \psi_\mu^i = (\nabla_\mu + \frac{1}{2} v_{\nu\rho} \gamma^{\nu\rho}_\mu - \frac{1}{3} v_{\nu\rho} \gamma_\mu \gamma^{\nu\rho}) \epsilon^i. \quad (2.9)$$

As mentioned above, because our background preserves R-symmetry the R-symmetry index just goes along for the ride. There is also a second fermion χ as well as gauginos whose variations determine the scalar auxiliary field D and field strengths, but turn out to not further constrain the Killing spinor and so shall not concern us here.

Let's first consider the $\delta\psi_r^i$ variation. There are two solutions with r -dependence $(r/l)^{\pm 1/4}$ and satisfying the projection $\gamma^{\hat{r}\hat{\theta}\hat{\phi}} \epsilon_\pm^i = \pm \epsilon_\pm^i$. Further, the two solutions are related by $\epsilon_-^i = (\sqrt{l/r}) \gamma^{\hat{+}\hat{r}} \epsilon_+^i$. Denote by ϵ^i the spinor satisfying $\gamma^{\hat{r}\hat{\theta}\hat{\phi}} \epsilon^i = \epsilon^i$. From the $\delta\psi_\pm^i$ variations we find that ϵ^i is independent of x^\pm , and that there is another solution of the form⁵

$$\lambda^i = -\frac{x^+}{l} \epsilon^i + \sqrt{\frac{l}{r}} \gamma^{\hat{+}\hat{r}} \epsilon^i. \quad (2.10)$$

Solving the angular variations for ϵ^i gives

$$\epsilon^i = \left(\frac{r}{l}\right)^{1/4} e^{\frac{\theta}{2} \gamma^{\hat{\theta}\hat{\phi}}} e^{-\frac{\phi}{2} \gamma^{\hat{\theta}\hat{\phi}}} \epsilon_0^i, \quad (2.11)$$

where ϵ_0 is a constant spinor which satisfies $\gamma^{\hat{r}\hat{\theta}\hat{\phi}} \epsilon_0^i = \epsilon_0^i$. In addition, the λ^i given in terms of ϵ^i in (2.10) remain solutions as well since these angular variations commute with $\gamma^{\hat{+}\hat{r}}$. So all in all we have 16 near horizon supersymmetries.

⁵ The λ^i are the enhanced supersymmetries of the near-horizon region. This equation expresses them in terms of the Lie derivative with respect to an $SL(2, \mathbf{R})$ Killing vector acting on ϵ^i .

2.3. Killing vectors

In order to determine the complete supergroup, we need to understand the action of the (right-handed) $SL(2, \mathbf{R})$ bosonic symmetries on the Killing spinors. The $SL(2, \mathbf{R})$ Killing vectors are

$$L_{-1} = l\partial_+, \quad L_0 = -x^+\partial_+ + r\partial_r, \quad L_1 = \frac{(x^+)^2}{l}\partial_+ - \frac{2x^+r}{l}\partial_r + \frac{4l^2}{r}\partial_-. \quad (2.12)$$

Using these we find that⁶

$$L_0\epsilon^i = \frac{1}{2}\epsilon^i, \quad L_0\lambda^i = -\frac{1}{2}\lambda^i, \quad L_1\epsilon^i = \lambda^i, \quad L_{-1}\lambda^i = -\epsilon^i, \quad (2.13)$$

which identifies ϵ^i and λ^i respectively with the $-\frac{1}{2}$ and $+\frac{1}{2}$ modes of G obeying $[L_m, G_r] = (\frac{m}{2} - r)G_{m+r}$.

Similarly the $SU(2)$ action is generated by

$$J_0^3 = -i\partial_\phi, \quad J_0^\pm = e^{\pm i\phi}(-i\partial_\theta \pm \cot\theta\partial_\phi). \quad (2.14)$$

Since $\gamma^{\hat{r}\hat{\theta}\hat{\phi}}$ and $\gamma^{\hat{\theta}\hat{\phi}}$ commute we can define

$$\gamma^{\hat{\theta}\hat{\phi}}\epsilon_0^i = \mp i\epsilon_0^i, \quad (2.15)$$

and it is easy to check that these satisfy

$$J_0^3\epsilon^i = \pm \frac{1}{2}\epsilon^i. \quad (2.16)$$

Suppose we start with a constant spinor obeying $\gamma^{\hat{\theta}\hat{\phi}}\epsilon_0 = -i\epsilon_0$ as well as $\epsilon_0 = -i\gamma^{\hat{0}\hat{\theta}}\epsilon_0^*$, and normalized to $\epsilon_0^\dagger\epsilon_0 = \frac{1}{4}$. Then we can define

$$\begin{aligned} \xi_-^1 &= \left(\frac{r}{l}\right)^{1/4} e^{\frac{\theta}{2}\gamma^{\hat{\phi}}} e^{-\frac{i}{2}\phi} (\gamma^{\hat{\theta}}\epsilon_0), \\ \xi_+^1 &= \left(\frac{r}{l}\right)^{1/4} e^{\frac{\theta}{2}\gamma^{\hat{\phi}}} e^{\frac{i}{2}\phi} \epsilon_0, \\ \xi_-^2 &= \left(\frac{r}{l}\right)^{1/4} e^{\frac{\theta}{2}\gamma^{\hat{\phi}}} e^{-\frac{i}{2}\phi} (-\gamma^{\hat{0}}\epsilon_0^*), \\ \xi_+^2 &= \left(\frac{r}{l}\right)^{1/4} e^{\frac{\theta}{2}\gamma^{\hat{\phi}}} e^{\frac{i}{2}\phi} (-\gamma^{\hat{0}\hat{\theta}}\epsilon_0^*), \end{aligned} \quad (2.17)$$

⁶ With the action defined via the Lie derivative $\mathcal{L}_K\epsilon = K^\mu\nabla_\mu\epsilon + \frac{1}{4}\partial_\mu K_\nu\gamma^{\mu\nu}\epsilon$.

where ξ^a is a **2** of $SU(2)$, $J_0^\pm \xi_\pm^a = 0$ and $J_0^\pm \xi_\mp^a = \xi_\pm^a$. We can organize these into symplectic-Majorana Killing spinors

$$\begin{aligned} \epsilon^{(1)} &= \begin{pmatrix} \xi_+^1 \\ -\xi_-^2 \\ 0 \\ 0 \end{pmatrix}, & \epsilon^{(2)} &= \begin{pmatrix} \xi_+^2 \\ -\xi_-^1 \\ 0 \\ 0 \end{pmatrix}, & \epsilon^{(3)} &= \begin{pmatrix} \xi_-^1 \\ \xi_+^2 \\ 0 \\ 0 \end{pmatrix}, & \epsilon^{(4)} &= \begin{pmatrix} -\xi_-^2 \\ -\xi_+^1 \\ 0 \\ 0 \end{pmatrix}, \\ \epsilon^{(5)} &= \begin{pmatrix} 0 \\ 0 \\ \xi_+^1 \\ -\xi_-^2 \end{pmatrix}, & \epsilon^{(6)} &= \begin{pmatrix} 0 \\ 0 \\ \xi_+^2 \\ -\xi_-^1 \end{pmatrix}, & \epsilon^{(7)} &= \begin{pmatrix} 0 \\ 0 \\ \xi_-^1 \\ \xi_+^2 \end{pmatrix}, & \epsilon^{(8)} &= \begin{pmatrix} 0 \\ 0 \\ -\xi_-^2 \\ -\xi_+^1 \end{pmatrix}, \end{aligned} \quad (2.18)$$

where each $\epsilon^{(I)}$ transforms as a **4** of $Sp(4)$ by left-multiplication (see appendix A). We will identify these with $G_{-\frac{1}{2}}^I$, $I = 1, \dots, 8$. Following the same procedure we can define

$$\eta_\pm^a = -\frac{x^+}{l} \xi_\pm^a + \sqrt{\frac{l}{r}} \gamma^{\hat{r}} \xi_\pm^a \quad (2.19)$$

and group them into symplectic-Majorana **4**'s which will be identified with $G_{\frac{1}{2}}^I$.

2.4. Supercharge commutators

Commutators of supercharges can be expressed as fermion bilinears involving the corresponding Killing spinors. In particular, [30] determines

$$\begin{aligned} \{G_r^I, G_s^J\} &\sim \Omega_{ij} \left((\bar{\epsilon}_r^{(I)})^i \gamma^\mu (\epsilon_s^{(J)})^j + (\bar{\epsilon}_s^{(J)})^i \gamma^\mu (\epsilon_r^{(I)})^j \right) \partial_\mu \\ &+ \left((\bar{\epsilon}_r^{(I)})_i \gamma^{\hat{\theta}\hat{\phi}} (\epsilon_s^{(J)})^j + (\bar{\epsilon}_s^{(J)})_i \gamma^{\hat{\theta}\hat{\phi}} (\epsilon_r^{(I)})^j \right), \end{aligned} \quad (2.20)$$

where $\epsilon_{-\frac{1}{2}}^{(I)} = \epsilon^{(I)}$ and $\epsilon_{\frac{1}{2}}^{(I)} = \lambda^{(I)}$. The first line of (2.20) involves the spacetime Killing vectors of $SL(2, \mathbf{R}) \times SU(2)$ and the second involves the generators of $Sp(4)$. Using our previous normalizations and the notation in appendix A, we find

$$\{G_{\pm\frac{1}{2}}^I, G_{\pm\frac{1}{2}}^J\} = -2\delta^{IJ} L_{\pm 1} \quad (2.21)$$

for $I, J = 1, \dots, 8$. Also,

$$\{G_{\frac{1}{2}}^{I_1}, G_{-\frac{1}{2}}^{J_1}\} = \begin{pmatrix} -2L_0 & -2iJ_0^3 + iA_3 & 2iJ_0^2 + iA_1 & 2iJ_0^1 + iA_2 \\ 2iJ_0^3 - iA_3 & -2L_0 & 2iJ_0^1 - iA_2 & -2iJ_0^2 + iA_1 \\ -2iJ_0^2 - iA_1 & -2iJ_0^1 + iA_2 & -2L_0 & -2iJ_0^3 - iA_3 \\ -2iJ_0^1 - iA_2 & 2iJ_0^2 - iA_1 & 2iJ_0^3 + iA_3 & -2L_0 \end{pmatrix}, \quad (2.22)$$

where $I_1, J_1 = 1, \dots, 4$. If $I_2, J_2 = 5, \dots, 8$, the same table arises with A_α replaced by C_α . If $I_1 = 1, \dots, 4$, and $J_2 = 5, \dots, 8$,

$$\{G_{\frac{1}{2}}^{I_1}, G_{-\frac{1}{2}}^{J_2}\} = \begin{pmatrix} iB_4 & iB_3 & iB_1 & iB_2 \\ -iB_3 & iB_4 & -iB_2 & iB_1 \\ -iB_1 & iB_2 & iB_4 & -iB_3 \\ -iB_2 & -iB_1 & iB_3 & iB_4 \end{pmatrix}. \quad (2.23)$$

These are just the commutation relations of $Os p(4^*|4)$, written below in a more compact form (assuming we rotate $G \rightarrow iG$):

$$\begin{aligned} \{G_r^I, G_s^J\} &= 2L_{r+s}\delta^{IJ} + (r-s)(t_\alpha)^{IJ}J_0^\alpha + (r-s)(\rho_A)^{IJ}R_0^A \\ [L_m, G_s^I] &= \left(\frac{m}{2} - s\right)G_{m+s}^I, \\ [R_0^A, G_r^I] &= (\rho^A)^{IJ}G_r^J \\ [J_0^\alpha, G_r^I] &= (t^\alpha)^{IJ}G_r^J, \end{aligned} \quad (2.24)$$

where t^α and ρ^A are the representation matrices for $SU(2)$ and $Sp(4)$ respectively, and R^A are the generators of $Sp(4)$. In the first two lines of (2.24), it should be understood we have only computed the global part of the superalgebra.

3. Towards an exact worldsheet CFT

In this section, we review and point out that the old results of [28] may be relevant to the problem of finding an exact worldsheet dual. We note that with the obvious adaptation of the GSO projection used in [28] one does not realize the needed 16 supercharges, so something more is needed to get a fully viable candidate for the worldsheet CFT.

3.1. 4d heterotic black monopoles

Heterotic string theory in four dimensions contains macroscopic black hole solutions [31] with magnetic charges lying in a $U(1)$ subgroup of $E_8 \times E_8$. Since the charges are associated with the left-moving sector of the worldsheet, such solutions are generically non-supersymmetric. The near horizon region is the product of 2D Minkowski space with a linear dilaton and an S^2 threaded with magnetic flux.

For every classical solution there should be a corresponding worldsheet CFT. In this case the CFT is rather subtle but was eventually found in [28]. While S^3 factors such as those arising in the near horizon for the NS5-brane are easily recognized as $SU(2)$

WZW models (which have $SU(2)_L$ and $SU(2)_R$ current algebras corresponding to the S^3 isometry group) it is harder to see where an S^2 horizon comes from (which has only one $SU(2)$ isometry). It turns out that it is given by an asymmetric orbifold of level $k = 2|Q^2 - 1|$ WZW model of the form

$$\frac{SU(2)_{2|Q^2-1|} \times SU(2)_{2|Q^2-1|}}{\mathbf{Z}_{2Q+2}}, \quad (3.1)$$

where Q is the monopole charge. (3.1) can be viewed as a two sphere with a left and a right fiber $U(1)_L$ and $U(1)_R$. The $U(1)_L$ fiber comes from the $U(1)$ subgroup of $E_8 \times E_8$ and the Chern class of the fibration is determined by the monopole charge. On the right, one has two fermions which are superpartners of the the two coordinates of the S^2 horizon and live in the tangent bundle. These can be bosonized to a $U(1)_R$ boson which also has a nontrivial fibration. The total space of the S^2 horizon together with its bosonized right-moving superpartners and the left-moving current $U(1)_L$ boson was shown in [28] to be given by (3.1), with a specified action for the \mathbf{Z}_{2Q+2} quotient.

3.2. 5d monopole-heterotic strings

We wish to consider two modifications of the construction of [28]. First, by trading a compact dimension for a trivial flat dimension, we can uplift the CFT to one describing a monopole string in five dimensions. Second, we replace the 3d $M^2 \times (\text{linear dilaton})$ factor with a $(0,1)$ $SL(2, \mathbf{R})_{k+4}$ WZW model⁷ (representing an AdS_3 factor) with the same central charges $(c_L, c_R) = (3 + \frac{6}{k+2}, \frac{9}{2} + \frac{6}{k+2})$ and constant dilaton.

The presence of H flux on the AdS_3 factor indicates that the monopole string also carries fundamental string charge. The number N of heterotic strings behaves as

$$N \sim \int_{S^2 \times M_5} e^{-2\Phi} * H \sim \frac{k}{g_5^2}. \quad (3.2)$$

We wish to have weakly coupled string theory so N must be large.

⁷ The supersymmetric right side contains bosonic level $k+4$ $SL(2, \mathbf{R})$ current j^A and a supersymmetric level $k+2$ $SL(2, \mathbf{R})$ current J^A .

3.3. $Q=0$: the heterotic string near-horizon

An intriguing feature of the construction of [28] is that it is nonsingular for the case $Q = 0$ which corresponds to $k = 2$. This case was referred to as the “neutral remnant” in [28]. $k = 2$ can be described by 3 left and 3 right free fermions, and the \mathbf{Z}_2 quotient in (3.1) acts purely on the left as a 2π rotation. One then expects that with the modifications of the previous subsection, the case $k = 2$ corresponds to the near-horizon geometry of N strings. However, to define the theory we must specify the GSO projection (with the $SL(2, \mathbf{R})$ factor there seems to be more than one way to do this), and the obvious adaptation of the one given in [28] does not give the needed spacetime supersymmetries. Possibly a different value of k^8 or modified GSO will give the desired theory.

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Appendix A. $Sp(4)$

An element $g \in Sp(4)$ satisfies $g^\dagger g = 1$ and $g^T \Omega g = \Omega$. If we parameterize g as e^{iM} , $M \in \mathfrak{sp}(4)$, then $M = M^\dagger$ and $M^T \Omega + \Omega M = 0$. This determines

$$\mathfrak{sp}(4) = \text{span}_{\mathbf{R}}\{A_1, A_2, A_3, B_1, B_2, B_3, B_4, C_1, C_2, C_3\}. \quad (\text{A.1})$$

Writing these in 2×2 blocks,

$$\begin{aligned} A_\alpha &= \begin{pmatrix} \sigma^\alpha & 0 \\ 0 & 0 \end{pmatrix}, \quad C_\alpha = \begin{pmatrix} 0 & 0 \\ 0 & \sigma^\alpha \end{pmatrix}, \\ B_\alpha &= \frac{1}{2} \begin{pmatrix} 0 & i^{\delta_{\alpha,2}} \sigma^\alpha \\ (i^{\delta_{\alpha,2}} \sigma^\alpha)^\dagger & 0 \end{pmatrix}, \quad B_4 = \frac{1}{2} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \end{aligned} \quad (\text{A.2})$$

where σ^α are the Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (\text{A.3})$$

⁸ The construction of [19] naively indicates that the value $k = 0$ (which gives bosonic $SL(2, R)$ currents at level 4), gives the desired spacetime central charge, but this construction requires modifications for a nonlinear superconformal algebra.

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